Numerical Analysis of Shielded Coplanar Waveguides

DAVID A. ROWE AND BINNEG Y. LAO

Abstract —**A numerical method is presented to calculate the impedance and the effective dielectric constant for a coplanar waveguide with a ground plane under a thin dielectric. Impedance and effective dielectric constant values are given as a function of geometry for GaAs and alumina substrates. An approximate analytical expression is given to generate the numerical value of the impedance with good accuracy. The results can be used in designing shielded coplanar waveguides. Measurements of a tapered 50-Ω test structure are presented.**

I. Introduction

THE USE OF integrated circuits at microwave frequencies requires transmission lines of a desired impedance provided directly to the vicinity of the IC chip. Coplanar waveguides on hybrid substrates are ideally suited for this application because of the coplanar ground plane furnished, and the possibility to taper the lateral dimensions from microstrip-line widths (20–50 mil) down to IC bonding pad dimensions (2–5 mil). Similar tapering of microstrip lines would require thinning of the dielectric, which sometimes is undesirable.

Practical hybrid substrates normally have a ground plane under the supporting dielectric. The presence of the ground plane increases the capacitance of the coplanar waveguide and thereby alters its impedance. When the lateral dimension B (Fig. 1) of the coplanar structure is much smaller than the dielectric thickness H, the effect of the ground plane is small and one approaches the standard coplanar waveguide limit. When B is much greater than H, capacitance to the lower ground plane dominates, and solutions for the pure microstrip case give a good approximation. The characteristic impedance for both limiting cases is well known analytically [1]–[4] and can be readily calculated for any given geometry. The intermediate case where B and H are comparable has been studied by various authors [5]–[8] using both quasi-TEM approximations and spectral domain analysis. Davies and Mirshekar-Syahkal [5] provide impedance data for a suspended dielectric above the ground plane. Another paper by the same authors [6] gives no impedance results and [7] gives no numerical data for the impedance. Shih and Itoh [8] compute the impedance by the spectral domain technique. In that approach, the computation time becomes significant (5 sec for 7 basis functions) for good accuracy when approaching the microstrip limit.

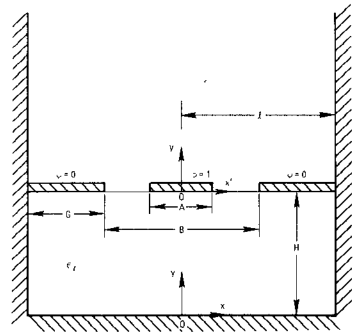

Fig. 1. Geometry of a shielded coplanar transmission line in a grounded box.

In this study, we present a solution to the shielded coplanar waveguide problem in the quasi-TEM limit. While it has been pointed out that the quasi-TEM approximation is not valid at high frequencies [6], the dispersion characteristics presented in [8] suggests that 1-percent accuracy in the effective dielectric constant can be maintained with this method up to 20 GHz.

The characteristic impedance can be calculated for any geometry in a short, 50-line, FORTRAN program. Each impedance calculation, accurate to the 100th order of the Fourier coefficient, takes 2 CPU sec on an IBM 3033. The accuracy is estimated to be better than 1 percent for most practical cases when A/B is less than 0.8. In order to compare the computational efficiency of this method, we have solved the identical problem by the relaxation method. The computation time required for the same accuracy is more than 20 CPU sec per data point.

The characteristic impedance for shielded coplanar waveguides on alumina and GaAs substrates is presented together with a closed-form expression that approximates the numerical results to 2-percent accuracy for most cases of practical importance. By using this approach, shielded coplanar waveguides can easily be designed.

II. Method of Analysis

The coplanar waveguide transmission line is imbedded in a grounded box as shown in Fig. 1. The problem then is to find the solution of Laplace's equation in the following two regions: the lower region (in the dielectric) with coordinate system (x, y), and the upper region (in air) with coordinate system (x', y'). The solution to Laplace's equation in the

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upper region takes on the form

$$\phi^{+}(x', y') = \sum_{i=1}^{\infty} a_i \cos(k_i x') \exp(-k_i y') \tag{1}$$

where $k_i = (2i-1)\pi/2l$ and $l = 0.5B + G$ as shown in Fig. 1. In the lower region

$$\phi^{-}(x, y) = \sum_{i=1}^{\infty} b_i \cos(k_i x) \sinh(k_i y). \tag{2}$$

The boundary condition at the surface of the dielectric leads to

$$a_i = b_i \sinh(k_i H). \tag{3}$$

In the gaps between the conductors, the surface-normal component of the displacement field is continuous. With the aid of (3), this implies

$$\sum_{i=1}^{\infty} a_i k_i \cos(k_i x)[1 + \epsilon_r \coth(k_i H)] = 0, \qquad \frac{A}{2} < x < \frac{B}{2} \tag{4}$$

where ϵ_r is the relative dielectric constant in the lower region. Without loss of generality, the voltage on the inner conductor is set to unity so that the conductor capacitance is numerically equal to the charge. This and the zero potential on the outer conductors lead to

$$\sum_{i=1}^{\infty} a_i \cos(k_i x) = \begin{cases} 1, & x \leqslant \frac{A}{2} \\ 0, & x \geqslant \frac{B}{2} \end{cases}. \tag{5}$$

To make the problem suitable for numerical analysis, the above Fourier sums are terminated at $i = N$. The problem is then to find the N unknown a_i's. To accomplish this, x takes on N discrete values $x = jl/(N+1)$, $j = 1, N$. Equations (4) and (5) for the a_i's can then be written as an N by N matrix equation

$$\sum_{i=1}^{N} m_{ji} a_i = d_j \tag{6}$$

where

$$m_{ji} = \begin{cases} k_i \cos(k_i x_j)[1 + \epsilon_r \coth(k_i H)], & \frac{A}{2} < x_j < \frac{B}{2} \\ \cos(k_i x_j), & x_j \geqslant \frac{B}{2} \text{ or } x_j \leqslant \frac{A}{2} \end{cases} \tag{7}$$

and

$$d_j = \begin{cases} 0, & x_j > \frac{A}{2} \\ 1, & x_j \leqslant \frac{A}{2} \end{cases}. \tag{8}$$

The matrix equation (6) is solved by Gaussian elimination to find the a_i's.

In order to compute the impedance, the capacitance per unit length from the inner conductor to ground is calculated by integrating the normal component of the displacement field on a Gaussian surface enclosing the inner conductor only. The capacitance is then

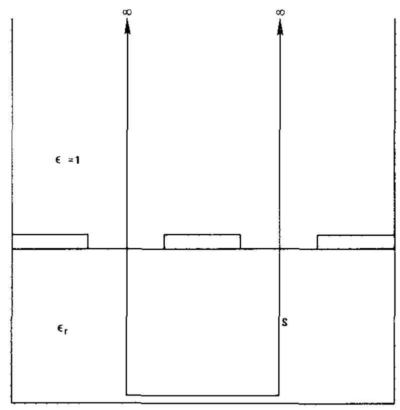

Fig. 2. Gaussian surface used in the capacitance calculation.

$$C = \epsilon_0 \oint_S \epsilon_r \vec{E} \cdot d\vec{S} = -\epsilon_0 \oint_S \epsilon_r \vec{\nabla}\phi \cdot d\vec{S}. \tag{9}$$

Fig. 2 shows the surface S chosen for this integration. This leads to the following equation for the capacitance:

$$C = -2\epsilon_0 \sum_{i=1}^{N} a_i \sin[k_i(A+B)/4][1 + \epsilon_r \coth(k_i H)]. \tag{10}$$

The static impedance and effective dielectric constant can be calculated from

$$Z_0 = \frac{1}{v\sqrt{CC_0}}, \qquad \epsilon_{\text{eff}} = \frac{C}{C_0} \tag{11}$$

where v is the velocity of electromagnetic propagation in vacuum and C_0 is the capacitance of the inner conductor to ground when the dielectric is removed. This capacitance can be calculated by the above technique by setting $\epsilon_r = 1$.

III. Results and Limitations

The calculated impedance for a shielded coplanar waveguide on alumina ($\epsilon_r = 9.9$) and GaAs ($\epsilon_r = 12.9$) is shown in Figs. 3 and 4, respectively. Values of ϵ_{eff} are also calculated for these cases as shown in Figs. 5 and 6. Note that the effective dielectric constant asymptotically approaches the theoretical value of $(\epsilon_r + 1)/2$ in the coplanar limit ($H/A \to \infty$).

The accuracy of this technique is limited by two factors. Firstly, the number of terms in the Fourier expansion is limited by the time required to perform the numerical inversion of the matrix. The computational time required for this operation increases as N^3 and effectively limits the matrix size.

Secondly, in most real situations, the sidewalls in Fig. 1 are not present. One would therefore like to make G sufficiently large to minimize the effect of the sidewall. Calculations show that $G = B/2$ is sufficient for 1-percent accuracy in impedance when $A/B \leqslant 0.8$ (less than 0.5-percent change in Z_0 was observed when G was increased by a factor of three).

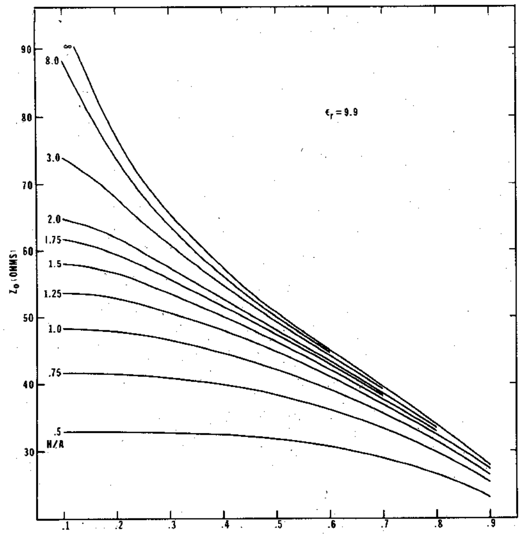

Fig. 3. Calculated characteristic impedance for shielded coplanar geometry on alumina ($\epsilon_r = 9.9$).

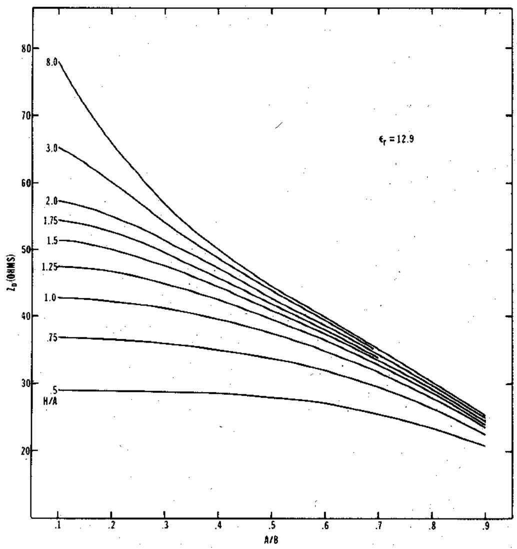

Fig. 4. Calculated characteristic impedance for shielded coplanar geometry on GaAs ($\epsilon_r = 12.9$).

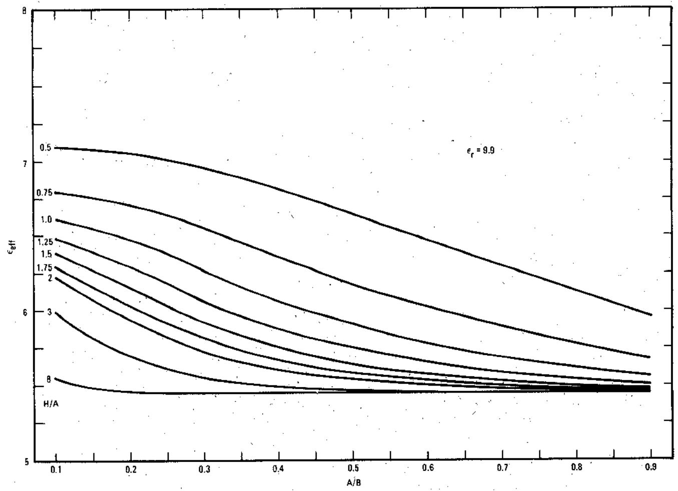

Fig. 5. Calculated effective dielectric constant for shielded coplanar geometry on alumina ($\epsilon_r = 9.9$).

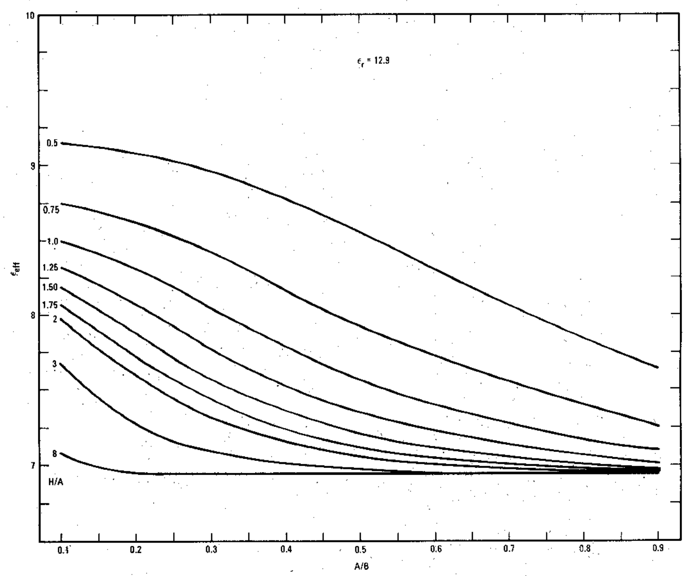

Fig. 6. Calculated effective dielectric constant for shielded coplanar geometry on GaAs ($\epsilon_r = 12.9$).

We have compared our results in the microstrip limit ($A/B \rightarrow 0$) and in the coplanar limit ($H/A \rightarrow \infty$) with published analytical solutions. For $\epsilon_r = 9.9$, the computed Z_0 in the microstrip limit agrees with Wheeler's result [2] to within 1 percent. The analytical prediction for Z_0 in the coplanar limit by Wen [1] is plotted in Fig. 3 as the $H/A = \infty$ curve. Our numerical results asymptotically approach this limit as $H/A \rightarrow \infty$. Comparison of results for $\epsilon_r = 12.9$ with those of Shih and Itoh [8] also shows good overall agreement. However, a 7-percent discrepancy was observed in the microstrip limit.

The results presented above can be used to design a tapered coplanar waveguide of constant impedance. In practice, one chooses an impedance and obtains B/A as a function of A/H. A typical case for a 50-Ω line on an alumina substrate ($\epsilon_r = 9.9$) is shown in Fig. 7. The tapering profile can then be designed in accordance with this figure.

The above analysis is two-dimensional and therefore does not give the effect of the rate of tapering on Z_0. However, an actual tapered test structure on $\epsilon_r = 4.7$ material was designed with a tapering angle of 20 degrees, as shown in Fig. 8. The reflection coefficient was measured by time-domain reflectometry as shown in Fig. 9 indicating the 50-Ω nature of the entire structure. The deviation in impedance can be accounted for by the measured 1.15-mil over-etch of the metal edges. It, therefore, seems that the effect of tapering on the impedance is not significant in this case.

To approximate the numerical results for Z_0 presented above, a closed-form empirical expression has been found for the shielded coplanar geometry shown in Fig. 1

$$Z_0 = \left[\frac{5q}{1+5q} \cdot \frac{1}{Z_m} + \frac{1}{1+q} \cdot \frac{1}{Z_c} \right]^{-1} \tag{12}$$

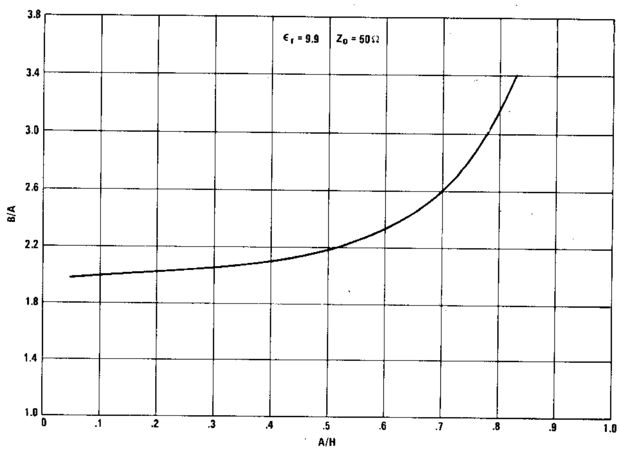

Fig. 7. Tapering profile for a 50-Ω line on alumina ($\epsilon_r = 9.9$).

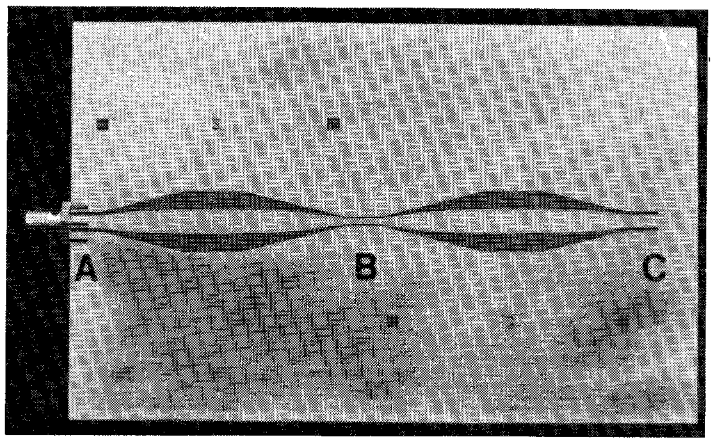

Fig. 8. Tapered shielded coplanar test structure on an $\epsilon_r = 4.7$ substrate. Line length is approximately 5 in and board thickness is 0.125 in. The line is terminated with a short.

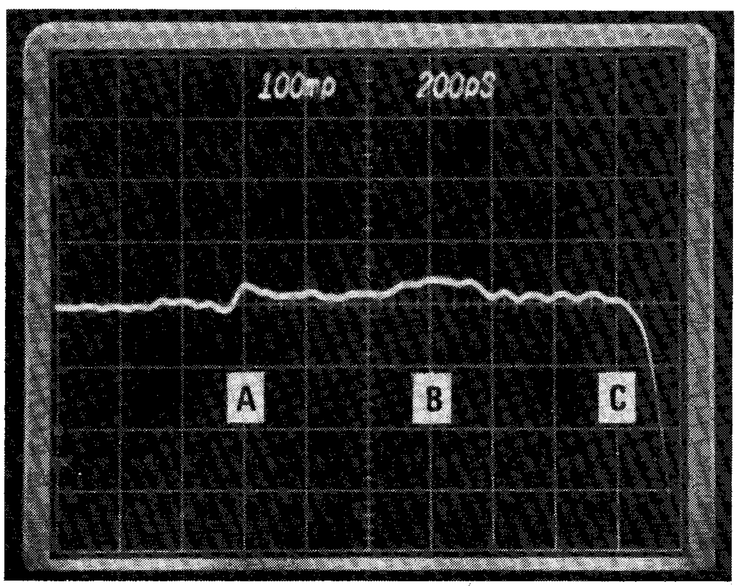

Fig. 9. Time-domain reflectometry measurement of the tapered test structure. The pulse used in the measurement has a rise time of 25 ps. The points A, B, and C refer to the locations on the structure shown in Fig. 8. Due to metal over-etching, the worst-case deviation from 50 Ω occurs where the line width is a minimum (B).

where

$$q=\frac{A}{H}\left(\frac{B}{A}-1\right)\left\{3.6-2\exp\left[-(\epsilon_r+1)/4\right]\right\}. \quad (13)$$

This function was constructed to reproduce the microstrip impedance Z_m exactly as q goes to infinity and similarly for the coplanar impedance Z_c as q goes to zero. Approximate expressions for Z_m and Z_c are given in [4, eqs. (3.52a, b) and (3.73)].

In the intermediate region, the accuracy of (12) for Z_0 has been checked against our numerical results for $0.25 \leqslant H/A \leqslant 3$, $0.1 \leqslant A/B \leqslant 0.8$, and $\epsilon_r = 1$, 5, 9.9, 12.9, 20, and 30. The worst-case discrepancy was found to be 2.2 percent. The average error for all points checked was 0.85 percent.

IV. Conclusion

The extensive use of MIC's makes it necessary to design coplanar waveguides on hybrid substrates having a nearby ground plane. In particular, a tapered coplanar waveguide could serve to bridge the dimensional difference that exists between the IC bonding pads and the widths of microstrip lines that serve as connections to other devices. A simple method is presented to calculate the impedance of coplanar waveguide structures with underside ground planes in the quasi-TEM limit. An analytical expression is provided to approximate the numerical results to about 2-percent accuracy. The speed and accuracy of the method makes it possible to design precision shielded coplanar waveguides of a desired impedance in an efficient manner.

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